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A NEW MODEL FOR PROBABILISTIC MULTI-PERIOD MULTI-OBJECTIVE PROJECT SELECTION PROBLEM

K. Khalili-Damghani[1], M. Poortarigh[2], A. Pakgohar[3]

[1]Department of Industrial Engineering, Tehran South Branch, Islamic Azad University, Tehran, Iran

[2]Economic Sciences University, Tehran, Iran

[3]Department of Finance, Accounting and Business Systems, Sheffield Business School, Sheffield, United Kingdom

Abstract

The project selection problem is considered as one of the most imperative decisions for investor organizations. Due to non-deterministic nature of some criteria in the real world projects in this paper, a new model for project selection problem is proposed in which some parameters are assumed probabilistic. This model is formulated as a non-linear, multi-objective, multi-period, zero-one programming model. Then the epsilon constraint method and an algorithm are applied to check the Pareto front and to find optimal solutions. A case study is conducted to illustrate the applicability and effectiveness of the approach, with the results presented and analysed. Since the proposed model is more compatible with real world problems, the results are more tangible and trustable compared with deterministic cases. Implications of the proposed approach are discussed and suggestions for further work are outlined.

Keywords:
Project selection, multi objective programming, chance constraint, epsilon constraint method.

1 INTRODUCTION

Decision making about selection or rejection of a project is considerable from both theoretical and practical aspects and it depends on satisfaction of financial and non-financial constraints of the problem. In the real world project selection problems there are normally more than one objective function. This makes the solution algorithm more complicated and time consuming.

The purpose of this paper is decision making about selection of N projects in T periods of time such that the profit gets maximum and total cost of equipment, human resources and used raw materials become minimum. In real world problems many of parameters are unlikely to be deterministic and ignoring the stochastic model can lead to unreliable results. In this paper a new framework is proposed for modeling a project selection problem relying on the concept of risk and by applying a linear approximation on probabilistic constraints. The mentioned model is called “Mean-Risk” model which the main idea of it is maximizing the expected profit of selecting a project such that risk curve of this selection always be below of the confidence curve. Here without loss of generality and just for simplification, the confidence is assumed a linear function.

Since some parameters are probabilistic, the constraints include these parameters are also probabilistic constraints. Chance constrained programming was developed as a means of describing constraints in the form of probability levels of attainment. Consideration of chance constraints allows the decision maker to consider objectives in terms of their attainment probability.

This approach changes constraints with stochastic parameters to constraints with a confidence level as the threshold of decision maker, using variance-covariance matrix. If α is a predetermined confidence level desired by the decision maker, the implication is that a constraint will have a probability of satisfaction of α.

After transforming the problem from probabilistic mode to deterministic mode, the resulting model is still multi objective and the optimal solution can be found on a Pareto front set. The method for solving multi objective problems used here is called augmented epsilon constraint method (AUGMECON). The AUGMECON method has been coded in Lingo 11, a widely used modeling language. The mentioned algorithm searches optimal solution by checking all points in the space of grid points while some of these points may be infeasible and algorithm wastes a lot of time by searching among infeasible points. More over by increasing the number of objective functions, the run time grows exponentially.

An innovative addition to the algorithm is the early exit from the nested loop when the problem becomes infeasible and there is no need to further restrict the corresponding objective function. This innovation significantly accelerates the algorithm in the case of several (more than three) objective functions. The rest of the paper is arranged as follows: Literature of past works which is presented in section 2 .In section 3 the problem will be illustrated, then the AUGMECON method will be introduced and an algorithm for this method will be presented. The results are shown in part 4. Finally the paper will be ended with conclusion remarks and further researches proposal in section 5.

2 LITERATURE OF PAST WORKS

Huang [1] introduced different types of mathematical programming to model a portfolio selection problem which there is no certainty the rate of return of investment and other parameters. He analyzed uncertainty under both conditions: variables with known distributions and parameters with unknown distributions. Defining risk curve and confidence curve, he presented Mean-Risk model to formulate portfolio selection problem with maximum utility regard to the risk curve achieved from combination of this selection which must be always under confidence curve that is under control of decision maker.

Rafiee et al. [2] analyzed and formulated the joint problem of project selection and project scheduling under uncertain environment. In this article a multi period project selection and scheduling problem is introduced and modeled by multi stage stochastic programming which is effective for solving long term planning problems under uncertainty. Assuming resources of the projects are limited and renewable, the aim was maximization of the present value of profits of projects as the objective function.

Olson et al. [3] presented a linear approximation for chance constrained programming in their article which can be used in either the single or multiple objective cases. The approximation presented will place a bound on the chance constraint at least as tight as the true nonlinear form, thus it overachieves the chance constraint at the expense of other constraints or objectives.

Mavrotas [4] made effort to effectively implement the epsilon constraint method for producing the Pareto optimal solutions in a MOMP. He proposed a novel version of the method AUGMECON that avoids producing weakly Pareto optimal solutions and accelerates the whole process by avoiding redundant iterations.

Atalay et al. [5] developed a method for transforming the chance constrained programming problem into a deterministic problem. In this article they considered the problem under this assumption that the random variables a_ij are independent with Gamma distribution. This new method used estimation of the distance between distributions of some of these independent random variables having Gamma or normal distributions. Probabilistic constraints obtained via Essen inequality which has been made deterministic using the approach suggested by Poly. The model studied on has been solved under the assumption of both Gamma and Normal distributions and the obtained results have been compared.

Khalili-Damghani et al. [6] developed a decision support system (DSS) to solve sustainable multi objective project selection problem with multi period planning horizon. First a TOPSIS based fuzzy goal programming (FGP) is proposed which considered uncertain DM preferences on priority of achievement level of fuzzy goals. The output of FGP and other affecting factors were treated as inputs of a fuzzy rule based system to estimate fitness value of an investment.

Pagnoncelli et al. [7] studied sample approximations of chance constrained problems. In particular they considered the sample average approximation (SAA) approach and discussed the convergence properties of the resulting problem and how good candidate solutions can be obtained by this method. In addition they presented a method for constructing statistical lower bounds for the optimal value of the considered problem and discussed how one should tune the underlying parameters.

Ekhtiari et al. [8] used multi objective stochastic programming to solve manpower allocation problem. They presented a novel combination of the chance constrained programming and the global criterion model for that which is called chance constrained global criterion. The proposed model is a deterministic equivalent for the multi objective problem of manpower allocation. To illustrate the model a tri-objective stochastic manpower allocation case problem for determining optimal number of manpower in a job-shop manufacturing system is formulated and solved and then the competitive advantages of the model were discussed.

Stochastic programming is one of the major approaches to deal with randomness or uncertainty involved in mathematical programming problems. Sakawa and. Kato [9] focused on multi objective linear programming problems with random variable coefficients in objective functions and/or constraints in his book. He used several stochastic models such as an expectation optimization model and a fractional criterion optimization model in chance constraint programming in order to transform the stochastic programming problems into deterministic ones.

Ackooij et al. [10] used joint chance constrained programming for hydro reservoir management in the paper. They presented an iterative algorithm for solving similarly structured joint chance constrained programming problems that requires a Slater point and the computation of the gradients. They also presented an individual chance constrained problem and a robust model. They illustrated the interest of chance constrained programming by comparing the results obtained by a realistic hydro valley with those obtained from the alternative models.

Ivanov [11] proposed a setting for a bi-level stochastic linear programming problem with quantile criterion and presented a deterministic equivalent of the problem for the case of the scalar random parameter. He showed an equivalent problem in the form of the two-stage stochastic discrete distribution of random parameters, the problem reduces to a mixed linear programming problem.

3 MATHEMATICAL PROGRAMMING MODEL

3.1 Problem description

There is T distinct independent periods of time and N projects in this problem. The decision maker must make decision about selection or rejection of a project such that all of objective functions will be optimized regard to the constraints, include resource and budget. Resources are involved in three groups of human, machine and raw material which are allocated independently in any period of time. The model is formulated as follows:

j	number of projects, j=1,2,…,n
i	type of human resources, i=1,2,…,m
k	machine kind, k=1,2,…,s
o	type of raw material, o=1,2,…,z

Parameters:

H_{it}	maximum available human resources of type I in period t p/h
h_{ij}	requirement of human resource i in project j (person-hour)
M_{kt}	maximum available machine-hour of type k in period t
m_{ij}	requirement of machine-hour of type k in project j
R_{ot}	maximum available raw material of type o in period t
r_{oj}	requirement of raw material o in project j
B_{jt}	maximum available budget for project j in period t
C_{it}	per hour cost of human resources I in period t
C_{kt}	per hour cost of machine type k in period t
C_{ot}	unit cost material o in period t
P_{jt}	total net profit or project j in period t
I_{jt}	Rate of return of project j in period t
d_{jt}	duration of project j in period t
$MARR_t$	minimum attractive rate of return in period t

The decision variable of the model is considered as below:

$$x_{jt} = \begin{cases} 1 & \text{if project j is selected for investment in period t} \\ 0 & \text{otherwise} \end{cases}$$

Eq. (1) tries to maximize the net profit of selected projects:

$$Max\ Z_1 = \sum_{t=1}^{T}\sum_{j=1}^{n} p_{jt} \times \mathrm{X}_{jt} \tag{1}$$

Eq. (2) tries to minimize the total cost of selected projects:

$$Min\ Z_2 = \sum_{t=1}^{T}\sum_{j=1}^{n} x_{jt} \sum_{i=1}^{m} h_{ij}.C_{it} + \sum_{t=1}^{T}\sum_{j=1}^{n} x_{jt} \sum_{k=1}^{s} m_{kj}.C_{kt} +$$

$$\sum_{t=1}^{T}\sum_{j=1}^{n} x_{jt} \sum_{o=1}^{z} r_{oj}.C_{ot} \tag{2}$$

Eq. (3) tries to maximize total internal rate of return of the selected projects:

$$Max\ Z_3 = \sum_{t=1}^{T}\sum_{j=1}^{n} x_{jt} \times I_{jt} \quad (3)$$

Eq. (4) the last objective function tries to minimize total unused resources of the optimum selections:

$$Min\ Z_4 = \sum_{t=1}^{T}[\sum_{i=1}^{m}(H_{it} - \sum_{j=1}^{n} h_{ij}.x_{jt}) + \sum_{k=1}^{s}(M_{kt} - \sum_{j=1}^{n} m_{kj}.x_{jt}) + \sum_{o=1}^{z}(R_{ot} - \sum_{j=1}^{n} r_{oj}.x_{jt})] \quad (4)$$

Constraints (5) which should be held for all projects express that each selected project must be selected only one time through the planning horizon:

$$\sum_{t=1}^{T} x_{jt} \leq 1, \qquad j = 1, 2, \dots, n \quad (5)$$

Constraints (6) which should be held for all projects present that each selected project must be completed in planning horizon:

$$\sum_{t=1}^{T}(t + d_{jt}).x_{jt} \leq T + 1, \qquad j = 1, 2, \dots, n \quad (6)$$

Constraints (7) which should be held for all human resources and planning horizons, insures that human resources availability is met during the procedure of project selection. The set of constraints (8) and (9) have the same description of constraints (7) but they are applied for machine-hour and raw materials, respectively:

$$\sum_{j=1}^{n} h_{ij} x_{jt} \leq H_{it}, \quad i = 1, 2, \dots, m, \quad t = 1, 2, \dots, T \quad (7)$$

$$\sum_{j=1}^{n} m_{kj} x_{jt} \leq M_{kt}, \quad k = 1, 2, \dots, T \quad t = 1, 2, \dots, T \quad (8)$$

$$\sum_{j=1}^{n} r_{oj} x_{jt} \leq R_{ot}, \quad o = 1, 2, \dots, z, \quad t = 1, 2, \dots, T \quad (9)$$

The set of constraints (10) , which should be held for all planning horizons and all projects, checks the budget availability in the project selection procedure:

$$\left(\sum_{i=1}^{m} h_{ij}.C_{it} + \sum_{k=1}^{s} m_{kj}.C_{kt} + \sum_{o=1}^{z} r_{oj}.C_{ot}\right) \times x_{jt} \leq B_{jt},$$
$$j = 1,2,\dots,n, t = 1,2,\dots,T \quad (10)$$

Constraints (11) checks if total cost of a selected project is less than its profit. They are also held for all projects in all planning horizons:

$$\left(\sum_{i=1}^{m} h_{ij}.C_{it} + \sum_{k=1}^{s} m_{kj}.C_{kt} + \sum_{o=1}^{z} r_{oj}.C_{ot}\right) \times x_{jt} \leq p_{jt},$$
$$j = 1,2,\dots,n, t = 1,2,\dots,T \quad (11)$$

Constraints (12) insure that the selected projects have a minimum internal rate of return equal to minimum attractive rate of return (MARR):

$$\sum_{j=1}^{n}\left(x_{jt}.(MARR_{t} - I_{jt})\right) \leq 0, \qquad t = 1, 2, \dots, T \quad (12)$$

Constraints (13) refer projects could be selected in each planning horizon:

$$\sum_{j=1}^{n} x_{jt} \geq 0, \qquad t = 1, 2, \dots, T \quad (13)$$
$$x_{jt} \in \{0,1\} \qquad j = 1, 2, \dots, n \qquad t = 1, 2, \dots, T$$

3.2 Mean-Risk Model

The main idea of this model is based on ensuring safety of the investment in the first priority, then maximizing or minimizing other objective functions. In the other word one should invest on a project if its risk curve is below of its confidence curve. The concept of confidence level for probabilistic condition is used. Here for simplification and without loss of generality the authors use a linear function as the confidence curve:

$$\alpha(r) = \begin{cases} cr + b & when\ k_1 \leq r \leq k_2 \\ cr + d & when\ k_2 \leq r \leq k_3 \\ e & when\ r \geq k_3 \end{cases} \quad (14)$$

Which r is a non-negative variable and represent the amount of loss, a, b, c, d and e are coefficients of function $\alpha(r)$.

$$R(r) = Pr\{r_f - (\xi_1 x_1 + \dots + \xi_n x_n) > r\}, \quad r \geq 0 \quad (15)$$

With above description, the mean-risk model will be formulated as follows:

Max E [$\xi_1 x_1 + \dots + \xi_n x_n$]
Subject to:
R ($x_1, x_2, \dots, x_n; r) \leq \alpha(r)$, $\forall r \geq 0$
$x_i \in \{0,1\} \quad i = 1, 2, \dots, n$ (16)

Now the project selection problem is re-written as a mean-risk model under this assumption that some parameters are probabilistic. So objective functions and constraints include these probabilistic parameters will change as follows:

Min E [Z_2]
Min E [Z_4]
Subject to:
Pr ($r_f - (-Z_2) \geq r) \leq \alpha(r)$, $\forall r \geq 0$
Pr ($r_f - (-Z_4) \geq r) \leq \alpha(r)$, $\forall r \geq 0$ (17)

As it is shown the second and forth objective functions which involve stochastic variables, changed and their expectation values are optimized. In addition, their corresponding constraints are added to the constraint set. Since z2 and z4 are minimization, their negative form must be put in the model. Other parts of the project selection problem model will stay unchanged.

3.3 Chance constraints

The constraints (7), (8), (9), (10), (11) include stochastic variables h_{ij}, m_{kj}, r_{oj} and are not solvable. To overcome this problem authors used chance constrained programming which uses variance-covariance matrix and modified the constrains as follows:

$$\sum_{j=1}^{n} h_{ij} x_{jt} + z\left[\sum_{j=1}^{n} v_{ij} x_{jt}^2\right]^{0.5} \leq H_{it}, \qquad i = 1, 2, \dots, m,$$
$$t = 1, 2, \dots, T \quad (18)$$

$$\sum_{j=1}^{n} m_{kj} x_{jt} + z\left[\sum_{j=1}^{n} v_{kj} x_{jt}^2\right]^{0.5} \leq M_{kt}, \quad k = 1, 2, \dots, s,$$
$$t = 1, 2, \dots, T \quad (19)$$

$$\sum_{j=1}^{n} r_{oj}\, x_{jt} + z\left[\sum_{j=1}^{n} v_{oj}\, x_{jt}^2\right]^{0.5} \leq R_{ot}\ , \quad o = 1,2,\dots,z,$$

$$t = 1,2,\dots,T \qquad (20)$$

$$\left(\sum_{i=1}^{m} h_{ij}\,.\,C_{it} + \sum_{k=1}^{s} m_{kj}\,.\,C_{kt} + \sum_{o=1}^{z} r_{oj}\,.\,C_{ot}\right) \times x_{jt} + z\left[\left(\sum_{i=1}^{m} v_{ij}\, C_{it}^2 + \sum_{k=1}^{s} v_{kj}\, C_{kt}^2 + \sum_{o=1}^{z} v_{oj}\, C_{ot}^2\right) x_{jt}^2\right]^{0.5} \leq B_{jt}$$

$$j = 1,2,\dots,n, t = 1,2,\dots,T \qquad (21)$$

$$\left(\sum_{i=1}^{m} h_{ij}\,.\,C_{it} + \sum_{k=1}^{s} m_{kj}\,.\,C_{kt} + \sum_{o=1}^{z} r_{oj}\,.\,C_{ot}\right) \times x_{jt} + z\left[\left(\sum_{i=1}^{m} v_{ij}\, C_{it}^2 + \sum_{k=1}^{s} v_{kj}\, C_{kt}^2 + \sum_{o=1}^{z} v_{oj}\, C_{ot}^2\right) x_{jt}^2\right]^{0.5} \leq p_{jt}$$

$$j - 1,2,\dots,n, \quad t - 1,2,\dots,T \qquad (22)$$

Moreover, two sets of constraints related to objective functions described in (17) will be added to the constrain set.

3.4 The augmented epsilon constraint method

The problem is considered as a multi objective, non-linear and zero one optimization problem. Generally in multi objective optimization problem there is no unique answer that can optimize all of objective functions simultaneously. Generally, a MODM problem with maximum objective functions can be formulated as follows:

$$\begin{aligned} &Max \quad (f_1(x), f_2(x), \dots, f_p(x)) \\ &St \qquad x \in S \end{aligned} \qquad (23)$$

In epsilon constraint method, decision maker optimizes one of the objective functions using the other functions as constraints, incorporating them in the constraint part of the model as shown below:

$$\begin{aligned} &Max \quad f_1(x) \\ &\text{st} \qquad f_2(x) \geq e_2 \\ &\qquad\quad f_3(x) \geq e_3 \\ &\qquad\quad \vdots \\ &\qquad\quad f_p(x) \geq e_p \\ &\qquad\quad x \in S \end{aligned} \qquad (24)$$

By parametrical variation in the RHS of the constrained (ei) the efficient solutions of the problem are obtained. The point of attention is that the optimal solution of problem (24) is guaranteed to be an efficient solution only if all the (p-1) objective function's constraints are binding. Otherwise if there are alternative optima, the obtained optimal solution of problem (34) is not in fact efficient, but it is a weekly efficient solution. In order to overcome this ambiguity the authors proposed the transformation of the objective function constraints to equalities by explicitly incorporating the appropriate slack or surplus variables. In the same time, these slack or surplus variables are used as a second term in the objective function, forcing the program to produce the efficient solutions. The new problem becomes:

$$\begin{aligned} &Max\ (f_1(x) + eps\,(s_2/r_2 + s_3/r_3 + \dots + s_p/r_p)) \\ &St \\ &f_2(x) - s_2 = e_2 \\ &f_3(x) - s_3 = e_3 \\ &\vdots \\ &f_p(x) - s_p = e_p \\ &x \in S \\ &s_i \in R^+ \end{aligned} \qquad (25)$$

Where eps is an adequately small number (usually between 10^{-3} and 10^{-6}. In order to avoid any scaling problem, it is recommended to replace the s_i in the second term of the objective function by ($^{s_i}/_{r_i}$), where r_i is the range of the ith objective function. Thus the objective function of the epsilon constraint method becomes like (25).

4 IMPLEMENTATION

The proposed model was applied in historical data of project selection of an Iranian financial and credit institution. The projects had been treated as investment chances. A set of four different investment chances including development of Public Transportation, Nano-Technology, Hybrid Electricity Power Plant and Refinery of Crude Oil were considered during a ten-year planning period (P1, P2, . . ., P10). Budgets, profit, internal rates, duration and MARR of investment chances are shown in Table1. The available resources for each project also depicted in Table 2. Due to the page limit of this paper the author is unable to provide other input data. However, full illustration will be presented at the conference session.

In this article it is assumed that the variables are independent and normally distributed. The objective functions have equal values for DM and there is no priority among them.

The Pareto border is divided into 4 distinct sections and algorithm runs with step length of 0.25. Epsilon is assumed (10^{-5}). The confidence function is linear and is variable from 0.2 to 0.9.

The results are shown in tables 3 and 4. The confidence curve is a continuous and decreasing function but in result's tables some special points of confidence curve which include the best solutions, were calculated. Regard to the data used in this article and the constraints, the model for confidence levels which are less than 0.82 will be infeasible. These results emphasize that such an investment involves a great deal of risk for investors. The models were coded by Lingo 11and the final solutions which are global optimal answers were obtained in different iterations of algorithm. It is obvious that in deterministic condition the objective functions have better results and gradually with increase of constraints in probabilistic condition, it gets worse.

5 CONCLUSION

In this paper, considering some factors and variables as stochastic variables, a new model for project selection problem was proposed. The structure of the traditional model was changed in both parts of objective functions and constraints which involved stochastic variables. In the objective function's part, risk-curve and confidence curve concept for ensuring the safety of the investment were used and in the constraints part, augmented chance constraint method for ensuring the feasibility of the problem was utilised. The main point in this research is determining an appropriate structure for confidence function which has a direct effect on the feasibility of problem and quality of solutions. Here, this structure is related to the investor (decision maker) but totally one needs exact information about the level of investment, market, business risks, economic factors and the type of investing to determine a certain confidence function. This point can be a crucial subject for future study. In addition, the proposed model was solved by epsilon constraint algorithm that all the results in both deterministic and probabilistic form were shown in the result's tables.

Table 1. Budgets, profit, internal rates, duration and MARR of investment chances.

Project	P1	P2	P3	P4	P5	P6	P7	P8	P9	P10
B_{jt} (10 Millions $)										
Public transportation	103,481	109,641	103,106	108,841	109,031	107,501	100,386	101,395	106,661	107,638
Nano-technology	131,525	131,910	143,952	136,524	113,575	135,271	109,027	128,180	144,436	139,199
Hybrid electricity power plant	121,241	101,068	113,213	108,301	111,135	101,216	103,496	116,041	115,269	106,526
Refinery of crude oil	109,321	100,958	102,040	111,229	111,145	104,946	101,504	112,461	115,741	112,085
P_{jt} (10 Millions $)										
Public transportation	34,494	36,547	34,369	36,280	36,344	35,834	33,462	33,798	35,554	35,879
Nano-technology	43,842	43,970	47,984	45,508	37,858	45,090	36,342	42,727	48,145	46,400
Hybrid electricity power plant	40,414	33,689	37,738	36,100	37,045	33,739	34,499	38,680	38,423	35,509
Refinery of crude oil	36,440	33,653	34,013	37,076	37,048	34,982	33,835	37,487	38,580	37,362
I_{jt} (%)										
Public transportation	5	7	6	8	6	7	7	7	7	8
Nano-technology	9	10	12	12	3	9	11	6	3	9
Hybrid electricity power plant	11	17	9	10	12	14	14	9	10	13
Refinery of crude oil	15	5	17	8	19	2	13	19	18	9
d_{jt} (years)										
Public transportation	2	2	2	3	2	1	3	3	1	2
Nano-technology	1	1	2	2	2	2	3	2	2	1
Hybrid electricity power plant	3	3	3	1	2	2	3	2	2	2
Refinery of crude oil	3	2	1	2	2	2	3	1	2	3
MARR	2	2	3	4	3	2	1	4	5	3

Table 2. Available resources of investment chances.

Human	P1	P2	P3	P4	P5	P6	P7	P8	P9	P10
H_{it}(man/period)										
Engineer	5000	5000	5000	5000	5000	5000	5000	5000	5000	5000
Technician	2000	2000	2000	2000	2000	2000	2000	2000	2000	2000
Manager	7000	7000	7000	7000	7000	7000	7000	7000	7000	7000
Worker	2500	2500	2500	2500	2500	2500	2500	2500	2500	2500

Machine	P1	P2	P3	P4	P5	P6	P7	P8	P9	P10
M_{kt}(quantity/period)										
Mechanic	8000	8000	8000	8000	8000	8000	8000	8000	8000	8000
Hydraulic/Pneumatic	4500	4500	4500	4500	4500	4500	4500	4500	4500	4500
Electric	4200	4200	4200	4200	4200	4200	4200	4200	4200	4200
Laboratory Equipment	8500	8500	8500	8500	8500	8500	8500	8500	8500	8500

Material	P1	P2	P3	P4	P5	P6	P7	P8	P9	P10
R_{ot}(m^3/period)										
Chemical	2000	2000	2000	2000	2000	2000	2000	2000	2000	2000
Water	3000	3000	3000	3000	3000	3000	3000	3000	3000	3000
Fossil/Sun energy	8000	8000	8000	8000	8000	8000	8000	8000	8000	8000
Mineral	1500	1500	1500	1500	1500	1500	1500	1500	1500	1500

Table 3. Payoff matrix of single objective optimization.

	Z_1	Z_2	Z_3	Z_4
Ideal calculations				
Max Z_1	162,021	14,851	38	331,838
Min Z_2	0	0	0	337200
Max Z_3	162,021	14,851	38	331,838
Min Z_4	162,021	14,851	38	331,838
Anti-ideal calculations				
Min Z_1	0	0	0	337200
Max Z_2	162,021	14,851	38	331,838
Min Z_3	0	0	0	337200
Max Z_4	0	0	0	337200

Table 4. Results of the probabilistic model with independent variables.

Selected projects	Z_1	Z_2	Z_3	Z_4	Confidence level
$X_{16}, X_{22}, X_{34}, X_{43}$	149917	13684	44	331838	0.95
X_{22}, X_{34}	80070	6657	20	334782	0.95
$X_{16}, X_{22}, X_{34}, X_{43}$	149917	13684	44	331838	0.9
X_{22}, X_{34}	80070	6657	20	334782	0.9
$X_{16}, X_{22}, X_{34}, X_{43}$	149917	13684	44	331838	0.85
X_{22}, X_{34}	80070	6657	20	334782	0.85
$X_{16}, X_{22}, X_{34}, X_{43}$	149917	13684	44	331838	0.82

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